Statistical Finance

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Research supported in part by grants from HKSAR-RGC-GRF

Overview

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Historical Development

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- Behavioral, Financial Econometrics, Active Risk Management, ... 2010 and beyond
- Statistical Finance, 2010 and forward. But what is it??

Overview

Static Model

- One-period investment, notable CAPM, which states that for a given portfolio $\mathbf{r} = (r_1, \ldots, r_p)$ and for a given μ, find $\mathbf{w} = (w_1, \ldots, w_p)$ such that

$$\min \quad \sigma_p^2 = \frac{1}{2}\mathrm{Var}(\mathbf{w}^{\mathrm{T}}\Sigma\mathbf{w}),$$

subject to

$$\mathrm{E}(\mathbf{w}^{\mathrm{T}}\mathbf{r}) = \mu \quad \text{and} \quad \sum_{i=1}^{p} w_i = 1,$$

where $\Sigma = \mathrm{Var}(\mathbf{r})$ and $\mathbf{w}$ are weights.

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- Factor Model
  Statistics comes into the picture
- Three-factor Model by Fama and French
- Arbitrage Pricing Theory ...

Dynamic Models

- Black-Scholes Model

$$
\begin{aligned}
dB_t &= \mu B_t \, dt \,, \\
dS_t &= \mu S_t dt + \sigma S_t \, dW_t, \\
d \log S_t &= \nu \, dt + \sigma \, dW_t,
\end{aligned}
$$

where W_t is the standard Brownian motion. A simple application of Itô's Lemma yields $\nu = \mu - \sigma^2/2$.

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- By taking $\mu = r$, the risk-free rate, and using a risk neutral argument, then any contingent claim on S_t, $C(S, t)$, can be shown to satisfy the Black-Scholes equation

$$\frac{\partial C}{\partial t} + (\frac{\partial C}{\partial S}) rS + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 = rC.$$

Dynamic Models

- From this equation, the celebrated Black-Scholes formula of an European call option

$$C(S) = S\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

can be derived, where $d_1 = \frac{1}{\sigma\sqrt{T}}\{\log(S/K) + (r + \sigma^2/2)T\}$ and $d_2 = d_1 - \sigma\sqrt{T}$. This formula was established in the late 70's and became a phenomenal success on the Wall Street.

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- The BS setup can be extended to many other claims
- But there are many shortcomings:
  - Parameter estimates, notably (σ)!
  - Why Brownian motions?
  - What about stochastic interest rates? ...

Each of these posts certain challenge, together huge challenges

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  There were articles blaming the BS for many of the financial disasters, particularly after the 2008 financial tsunami. Eventually, this leads to financial reforms in the US market, in particular the so-called Dodd-Frank Wall Street Reform and Consumer Protect Act, signed into law on July 21, 2010, is a response to the calling for regulatory reform after the 2007–08 financial crisis. The provisions of the law include:

Dynamic Models

- Financial stability, which establishes the Financial Stability Oversight Council (FSOC) and the Office of Financial Research (OFR) to monitor systemic risk and identify risks to the US financial system.
- Collect data and conduct research, promote market discipline.
- Orderly liquidation authority for insured depository institutions and securities by the FDIC or SIPC (Security Investor Protection Corporation).
- Regulation of hedge funds, and similar investment intermediaries.
- Insurance reform which establishes the Federal Insurance Office to monitor the insurance industry (except health insurance), especially gaps in the regulation of insurers that could contribute to financial crisis.
- Volcker rule, ...

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- Dynamic factor model and the Bayesian paradigm, mainly empirical Bayes idea by Lai et al. (2006)
- Volatility analysis

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Dynamic Model

- Go beyond the Black-Scholes world
- $W_t \to L_t$, a Levy process which accounts for jumps
- Modeling of σ_t, ARCH-GARCH type. Apply that to BS, Duan (90's). That uses statistics
- Extend W_t to W_t^H, the fractional Brownian motion. The main caveat W_t^H is not a martingale in general, hence the no arbitrage argument fails. One needs to make use of Malliavin calculus. Chan and Ng (2006 and 2009) and Buchmann and Chan (2009)

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  - Too theoretical, not enough interest from practitioners.
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- Potentials: High if applied correctly.

American-Type and Exotic Options

- Free-boundary value problem of a SDE, dynamic programming solutions
- Heavy computational burdens, parallel computing
- Heavy simulations
- Exotic Options such as Asian Option, Barrier Options, ... Many recent developments, Hull (2014)

- Fixed Income and Structure Products
- Mostly Black-Scholes based
- Mainly option-type (embedded)

Overview

More Recent Developments

Time Series

- Consider the time series r_t, the return at the end of period t
- $r_t = \beta r_{t-1} + \epsilon_t$, AR(1), $\beta = 1$, unit-root model
- Random walk model, related to the efficient market hypothesis (EMH)

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- Unified asymptotic theory, Chan (2006, 2010, 2015 ...)

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- Notable progresses, statistical arbitrage, high-frequency data and market micro-structures, statistical trading strategies, active risk management (active health care), Lai and Xing (2008) and Chan (2010).

Systemic Risk

- Systemic risk is the risk of a broad-based breakdown in the financial system, often realized as a series of correlated defaults among financial institutions, typically banks.
- The events of 2007–2009 have shown that bank-type runs can affect non-bank entities as well, such as money market funds, insurance companies and hedge funds. Some characteristics: liquidity dried up, sudden change in regime such as extreme volatility, heightened uncertainty, etc.
- Systemic shocks come from many directions: sovereign default that sank the LTCM, sub-prime mortgage crisis, algo feedback loop of ash crash on May, 2010, etc.
- The Dodd-Frank Act and ongoing research by the IMF and ECB (European Central Bank) attempt to mitigate systemic risk.

Networks

- Networks have pervasive applications (the internet biological/economic/engineering/social networks). Most of the network models studied are fixed exogenously, but financial networks of interest to systemic risk modeling are endogenous, involving profit-maximizing financial institutions.
- Core-periphery network structure: A few core (big) banks linked to (and providing liquidity for) smaller banks in the network.
- Failure propagation in the network: A way to evaluate the vulnerability of a system is to perform simulations and stress tests (codified in the Basel Accord).
- How the network changes in a financial crisis, and how to response to these changes?

Literature Review

- On Financial Networks and Systemic Risk Billio, Getmanksi, Lo, Pellizzon (2012), Diebold and Yilmaz (2011), Hautsch, Schaumburg, Schienle (2011), Bisias, Flood, Lo, Valavanis (2012), Brunnermeier and Oehmke (2012)
- On Graphical Models Dempster (1972), Lauritzen (1996), Meinshausen and Buhlmann (2006)
- On lasso Estimation Tibshirani (1996), Fan and Peng (2004), Zou (2006), Peng, Wang, Zhou, Zhu (2009)
- On Sparse Covariance Estimation Bickel and Levina (2008), Lam and Fan (2009), Fan, Liao, Mincheva (2013)
- On Sparse Spectral Density Dahlhaus (2000), Eichler (2007), Davis, Zang, Zheng (2012)
- On Robust Covariance Estimation White (1984), Gallant (1987), Newey and West (1991), Andrews (1991), Andrews and Monahan (1992), Den Haan and Levin (1994)

Operational Risks

- Operational risk is the risk of loss caused by inadequate or failed internal processes, people and systems, or external events. The loss ascribed to operational risk may also be due to using a wrong model to value trading positions (called "model risk") or technical errors, inadequate supervision of trade execution, fraud, or accident. As pointed out by Jorion (2006), "traders using a conventional option pricing model, for instance, could be exposed to model risk if the model is misspecified or if the model parameters are erroneous," and "to guard against model risk, models must be subjected to independent evaluation using market prices, when available, or objective out-of-sample evaluations."
- An important component of operational risk is related to the corporate structure, which is itself a network, of the financial institution. There are divergent utilities of the utility-maximizing agents at different nodes of this network. Their aggregation can contribute to systemic risk, e.g. in the housing bubble.

Operational Risks (continued)

- An example is that when the housing market appeared unsustainable and data showed growing delinquencies in 2006, many banks were still promoting sub-prime mortgages. Employees in the mortgage division had to keep putting their bets to get bonuses and avoid losing their jobs. The Dodd-Frank Act contains a whistle blowing provision and a claw back provision to mitigate such risk.

Active Risk Management

- "One cannot manage what one cannot measure. A prerequisite for effective financial regulatory reform is to develop dedicated infrastructure for defining, measuring, monitoring, and investigating systemic risk on a standardized, ongoing, and regular basis." (Lo, 2009 written testimony to the U.S. House of Representatives). The infrastructure should involve distributed data updating and analysis together with centralized statistical modeling and sequential monitoring to support early warning and timely decisions to manage risk actively.
- We borrow the adjective "active" in risk management from "active portfolio management" introduced by Grinold and Kahn. It means dynamic and adaptive, with continual adjustments/rebalancing, to strike a suitable balance between risk and return.

Active Risk Management (continued)

- Active risk management applies to different nodes of a hierarchical financial network, ranging from different units (culminating in senior management) of a bank/firm to the supervisory/regulatory hierarchy involving such entities as the FED and the ECB. It is similar to "total quality management" in the manufacturing industry, for which statistical tools have been developed in the past 50 years. These tools can be modified and combined with financial/economic models for active risk management in financial networks.

Statistical Process Control, SPC

- SPC is arguably a misnomer for "statistical process monitoring" and represents a collection of techniques in detecting deviations of a production process from a "state of statistical control" and taking corrective actions to restore that state. The techniques typically take the form of quality control charts, and SPC was called SQC in the early literature.
- The control charts involving sampling inspections in industrial engineering have evolved into real-time fault detection schemes in aeronautical and electrical engineering. The sequential detection schemes can be formulated as stopping rules, and a theory of optimal stopping to achieve quickest detection subject to a prescribed constraint on the false alarm rate has been developed (Lai, 1995, JRSSB).

Statistical Process Control

- Sequential surveillance is a modification of sequential detection methodology for the case where corrective actions may not be taken immediately after detection because of operational constraints. Thus there may be multiple change-points before action is finally taken.
- Estimation and change-point detection in regime-switching and more general change-point models: HMMs (Lai and Xing: bounded complexity mixture filters, 2012, 2013).
- Extension of existing theory to financial networks: ongoing research.
- Early warning: Involves detection of change-points for nodes of a network (e.g., the node is a bank and the edges represents the counter-parties or loan holders) or for predictive covariates (e.g., macroeconomic variables, equity prices). It also involves transforming multivariate (potentially high-dimensional) outcome variables and their predictors into low-dimensional early warning indicators. This is a very active area of research, especially at central bank level for early warning indicators of systemic risk.

Modifications to Financial Risk Management

- The first step is to extend the preceding methodology to networks.
- A major modification for financial networks is related to their hierarchical structure, and more importantly, that such networks involve people rather than machines. Thus, the network has already natural built-in adaptation and control components but may have multiple objective functions (sometimes conflicting) for multiple economic agents that can lead to "systemic" failures. This is an ongoing research project.
- Small, gradual changes as in stochastic adaptive control work much better than sudden, drastic corrective actions in human communities, financial institutions, government oversight and macroeconomic policies.

Conclusion

- An active risk management approach is useful not only to avoid financial bubbles but also to counter the cavalier trading risk for short-term profit.
- Deming's SQC revolutionized quality in the Japanese manufacturing industry in the 1960s. Perhaps the closely related sequential surveillance and adaptive risk control methodologies can lead to innovations for financial markets.
- Taleb's "black swan" symbolizes the futility of prediction in financial markets. It is virtually impossible to foresee a black swan until it appears. On the other hand, the "highly improbable" events he describes can be linked to "black-necked" swans instead, and active risk management can inhibit their occurrence or at least mitigate their impact.
- In light of the gain-maximization rationale on the Wall Street that drives financial practitioners ever closer to their ethical boundaries, the market needs practitioners to be scientifically critical, socially honest and adherent to the utmost ethical standards to resist temptations.
- Statistical Finance may be perhaps a means to facilitate these goals.

Conclusion

Thank You!!